Q. No. 2 Part (i)

Verify $\omega^{40} + \omega^{50} + 1 = 0$

Solution:-

$L.H.S. = \omega^{40} + \omega^{50} + 1$

$= (\omega^{39}) \times \omega + \omega^{50} + 1$

$= (\omega^{3})^{13} \times \omega + \omega^{48} \times \omega^{2} + 1$

$= (1)^{13} \times \omega + (\omega^{3})^{16} \times \omega^{2} + 1 \qquad \because \omega^{3} = 1$

$= 1 \cdot \omega + 1 \cdot \omega^{2} + 1$

$= \omega + \omega^{2} + 1$

$= 0 = R.H.S. \qquad \because \omega + \omega^{2} + 1 = 0$

Hence:

L.H.S. = R.H.S. (proved)

Q. No. 2 Part (ii)

$U = \{1, 2, 3, \ldots, 10\}$

$P = \{2, 3, 4, 5, 8\}$

$Q = \{1, 3, 5, 7, 9\}$

Verify $(P - Q)' = P' \cup Q$

Solution:-

L.H.S. $= (P - Q)'$

$P - Q = \{2, 3, 4, 5, 8\} - \{1, 3, 5, 7, 9\}$

$= \{2, 4, 8\}$

$(P - Q)' = U - (P - Q)$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 8\}$

$= \{1, 3, 5, 6, 7, 9, 10\}$

R.H.S. $= P' \cup Q$

$P' = U - P$

$= \{1, 2, 3, 4, \ldots, 10\} - \{2, 3, 4, 5, 8\}$

$= \{1, 6, 7, 9, 10\}$

$P' \cup Q = \{1, 6, 7, 9, 10\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 3, 5, 6, 7, 9, 10\}$

L.H.S. = R.H.S.

$(P - Q)' = P' \cup Q$ (proved)

Q. No. 2 Part (iii)

Verify $\dfrac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$

Solution :-

$$L.H.S. = \frac{1}{\sec\theta - \tan\theta}$$

Multiplying numerator and denominator by $\sec\theta + \tan\theta$:

$$= \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{(\sec^2\theta - \tan^2\theta)} \qquad \therefore (a-b)(a+b) = a^2 - b^2$$

$$= \frac{\sec\theta + \tan\theta}{1} \qquad \therefore 1 + \tan^2\theta = \sec^2\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

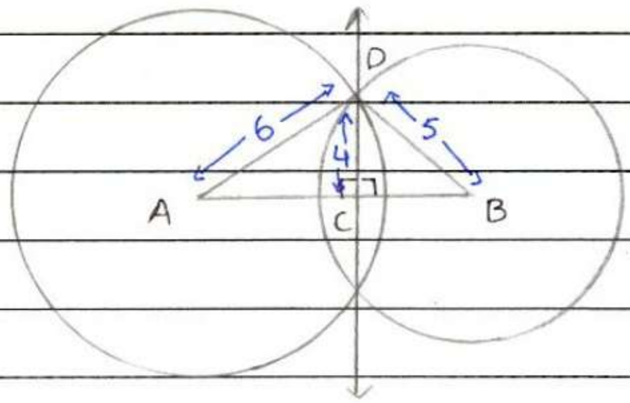
$= \sec\theta + \tan\theta = R.H.S.$

$L.H.S. = R.H.S.$

$$\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta \quad \text{(proved)}$$

Q. No. 2 Part (iv)

Figure :-



Given Data:-

$m\overline{AD} = 6cm$, $m\overline{CD} = 4cm$

$m\overline{BD} = 5cm$, $m\overline{AB} = ?$

Solution:-

Consider $\angle rt\ \Delta\ ACD$:

By Pythagoras: Theorem: $(hyp)^2 = (base)^2 + (perp)^2$

$(6)^2 = (m\overline{AC})^2 + (4)^2$

$36 - 16 = (m\overline{AC})^2$

$m\overline{AC} = \sqrt{20} = 4.47$

Now consider $\angle rt\ \Delta\ BCD$:

$(hyp)^2 = (base)^2 + (perp)^2$

$(5)^2 = (m\overline{BC})^2 + (4)^2$

$25 - 16 = (m\overline{BC})^2$

$m\overline{BC} = \sqrt{9} = 3$

$m\overline{AB} = m\overline{AC} + m\overline{BC}$

$= 4.47 + 3$

$\boxed{m\overline{AB} = 7.47\ cm}$

Distance between centres A and B is 7.47 cm.

Q. No. 2 Part (v)

α, β are roots of $x^2 - 3x + 6 = 0$

Form equation with roots $\alpha^2\beta, \alpha\beta^2$

Solution:-

$\alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3$

$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

Given roots $= \alpha^2\beta, \alpha\beta^2$

Sum $= \alpha^2\beta + \alpha\beta^2$

$= \alpha\beta(\alpha + \beta)$

$= 6(3)$

$= 18$

Product $= \alpha^2\beta \times \alpha\beta^2$

$= \alpha^3\beta^3 = (\alpha\beta)^3$

$= (6)^3$

$= 216$

As $x^2 - Sx + P = 0$

$\boxed{\text{Equation: } x^2 - 18x + 216 = 0}$

Q. No. 2 Part (vi)

Solve $4(2^{2x+1}) - 9(2^x) + 1 = 0$

Solution:-

$4 \cdot 2^{2x} \cdot 2^1 - 9 \cdot 2^x + 1 = 0$

let $2^x = y$

$\Rightarrow (y)^2 = (2^x)^2$

$y^2 = 2^{2x}$

Now: $4 \cdot y^2 \cdot 2 - 9 \cdot y + 1 = 0$

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(y-1)(8y-1) = 0$

$y - 1 = 0$, $8y - 1 = 0$

$y = 1$, $y = 1/8$

$2^x = 1$, $2^x = \frac{1}{8}$

$2^x = 2^0$

comparing exponents: , $2^x = 2^{-3}$

$\boxed{x = 0}$, comparing exponents:

, $\boxed{x = -3}$

Solution set = $\{0, -3\}$

Q. No. 2 Part (vii)

$\sin\theta = \frac{-2}{3}$, terminal arm of θ is in IV quadrant.

Solution:-

$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{y}{r} = \frac{-2}{3}$

$\Rightarrow \quad y = -2 \quad , \quad r = 3$

By Pythagoras Theorem:

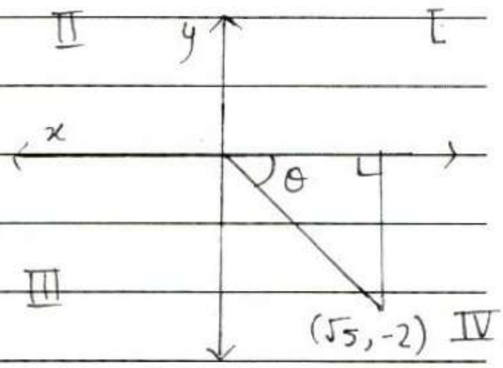
$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$

$(3)^2 = (x)^2 + (-2)^2$

$9 - 4 = x^2$

$x = \sqrt{5}$

Figure:



Remaining Trigonometric Values:

$\csc\theta = \frac{r}{y} = -\frac{3}{2}$

$\cos\theta = \frac{x}{r} = \frac{\sqrt{5}}{3}$

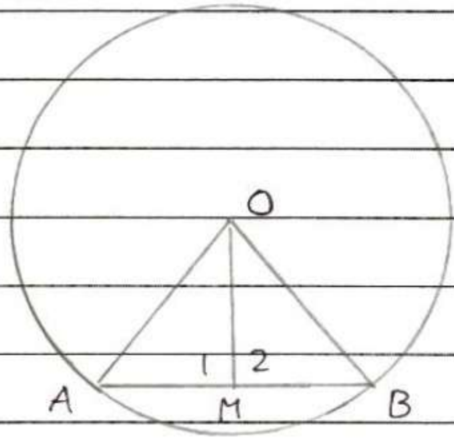
$\sec\theta = \frac{r}{x} = \frac{3}{\sqrt{5}}$

$\tan\theta = \frac{y}{x} = \frac{-2}{\sqrt{5}}$

$\cot\theta = \frac{x}{y} = -\frac{\sqrt{5}}{2}$

Q. No. 2 Part (viii)

Theorem: "Prove that straight line ....."



Given:

$\overline{AB}$ is a chord of circle with centre O. $\overline{OM}$ bisects chord $\overline{AB}$ so $m\overline{AM} = m\overline{BM}$

To Prove:

$m\overline{OM} \perp$ chord $\overline{AB}$

Construction:

Join O to A and B to form Δs OAM and OBM.

Proof:-

Statements	Reasons
In $\Delta OAM \longleftrightarrow OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of same circle
$m\overline{OM} = m\overline{OM}$	common
$m\overline{AM} = m\overline{BM}$	given
$\Delta OAM \cong \Delta OBM$	S.S.S. $\cong$ S.S.S.
$m\angle 1 = m\angle 2$ .....i)	corresponding $\angle$s of congruent Δs.
$m\angle 1 + m\angle 2 = 180°$ ....ii)	adjacent supplementary $\angle$s
$m\angle 1 = m\angle 2 = 90°$	from i) and ii)
i.e. $m\overline{OM} \perp m\overline{AB}$.	

Q. No. 2 Part (ix)

Class	Frequency	Midpoints (x)	f / x	
31 - 40	28	35.5	0. 78	
41 - 50	18	45.5	0.39	
51 - 60	10	55.5	0.18	
61 - 70	14	65.5	0.21	
71 - 80	30	75.5	0.39	
	$\Sigma f = 100$		$\Sigma(f/x) = 1.95$	

$$\text{Harmonic mean} = \frac{\Sigma f}{\Sigma(f/x)}$$

$$= \frac{100}{1.95} = 51.28$$

$$\boxed{\text{Harmonic mean} = 51.28}$$

Q. No. 3 (Page 1)

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$P = \{2, 4, 6, 8, 10\}$

$Q = \{2, 3, 5, 7\}$

Solution :-

DE MORGAN's LAWS:

(i): $(A \cup B)' = A' \cap B' \Rightarrow (P \cup Q)' = P' \cap Q'$

$L.H.S = (A \cup B)' = (P \cup Q)'$

$P \cup Q = \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 7\}$

$= \{2, 3, 4, 5, 6, 7, 8, 10\}$

$(P \cup Q)' = U - (P \cup Q)$

$= U - \{2, 3, 4, 5, 6, 7, 8, 10\}$

$= \{1, 2, 3, \ldots, 10\} - \{2, 3, 4, 5, 6, 7, 8, 10\}$

$= \{1, 9\}$

$R.H.S. = A' \cap B' = P' \cap Q'$

$P' = U - P = \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$Q' = U - Q$

$= \{1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$P' \cap Q' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{1, 9\}$

Hence,

$L.H.S. = R.H.S.$

$(P \cup Q)' = P' \cap Q'$ (proved)

Q. No. 3 (Page 2)

(ii): $(A \cap B)' = A' \cup B' \Rightarrow (P \cap Q)' = P' \cup Q'$

L.H.S. $= (A \cap B)' = (P \cap Q)'$

$P \cap Q = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 7\}$

$= \{2\}$

$(P \cap Q)' = U - (P \cap Q)$

$= \{1, 2, 3, \ldots, 10\} - \{2\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$

R.H.S. $= A' \cup B' = P' \cup Q'$

$P' = U - P$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$Q' = U - Q$

$= \{1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$

$= \{1, 4, 6, 8, 9, 10\}$

$P' \cup Q' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9, 10\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$

Hence,

L.H.S. = R.H.S.

$(P \cap Q)' = P' \cup Q'$ (proved)

Q. No. 4 (Page 1)

$x^2 + y^2 = 20$

$6x^2 + xy - y^2 = 0$

Solution:-

$x^2 + y^2 = 20$ —— ①

$6x^2 + xy - y^2 = 0$

$6x^2 + 3xy - 2xy - y^2 = 0$

$3x(2x+y) - y(2x+y) = 0$

$(2x+y)(3x-y) = 0$

$2x + y = 0$, $3x - y = 0$

$y = -2x$ —— ⑪ , $y = 3x$ —— ⑪

Put value of y in ① , Put value of y in ①

$x^2 + (-2x)^2 = 20$, $x^2 + (3x)^2 = 20$

$x^2 + 4x^2 = 20$, $x^2 + 9x^2 = 20$

$x^2 + 4x^2 - 20 = 0$, $10x^2 = 20$

$5x^2 - 20 = 0$, $x^2 = \frac{20}{10}$

$5x^2 = 20$,

$x^2 = 4$, $x^2 = 2$

$x = \sqrt{4}$, $\boxed{x = \pm\sqrt{2}}$

$\boxed{x = \pm 2}$ Put value of x in ⑪

Put value of x in ⑪ ,

$y = 3(\pm\sqrt{2})$

$\boxed{y = \pm 3\sqrt{2}}$

$y = -2x$

$y = -2(\pm 2)$

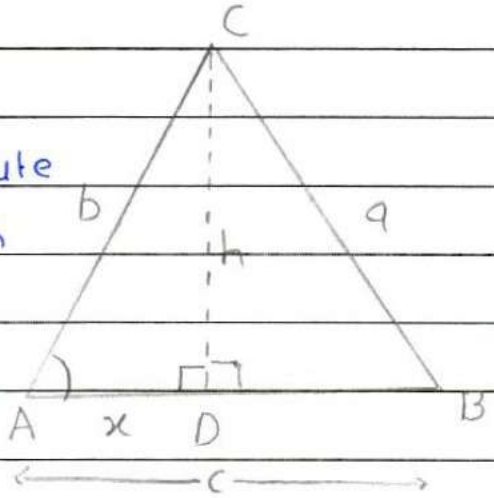
$\boxed{y = \pm 4}$

Solution set = $\{(\pm 2, \pm 4), (\pm\sqrt{2}, \pm 3\sqrt{2})\}$

Q. No. 4 (Page 2)

Q. No. 5 (Page 1)

Theorem: "Prove that in any triangle ...... projection on it of the other."



Given:

ΔABC is a triangle with an acute angle $\angle CAD$ at A. $\overline{CD}$ is drawn perpendicular to $\overline{AB}$, ~~so~~ and that $\overline{AD}$ is the projection of $\overline{AC}$ on $\overline{AB}$.

Let $m\overline{AC} = b$, $m\overline{AB} = c$, $m\overline{BC} = a$, $m\overline{AD} = x$ and $m\overline{DC} = h$.

To Prove:

$$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$$

i.e. $a^2 = b^2 + c^2 - 2cx$

Proof:-

Statements	Reasons
In $\angle$rt ΔCDA $(\overline{AC})^2 = (\overline{AD})^2 + (\overline{DC})^2$ $b^2 = x^2 + h^2 \quad \ldots i)$	By Pythagoras theorem
Now in $\angle$rt ΔCDB $(\overline{BC})^2 = (\overline{DB})^2 + (\overline{DC})^2$ $a^2 = (c-x)^2 + h^2$	By Pythagoras theorem

Q. No. 5 (Page 2)

$a^2 = c^2 - 2cx + x^2 + h^2$

$a^2 = c^2 - 2cx + (x^2 + h^2)$

$a^2 = c^2 - 2cx + b^2$ from i) $b^2 = x^2 + h^2$

i.e. $a^2 = b^2 + c^2 - 2cx$

or,

$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$ proved